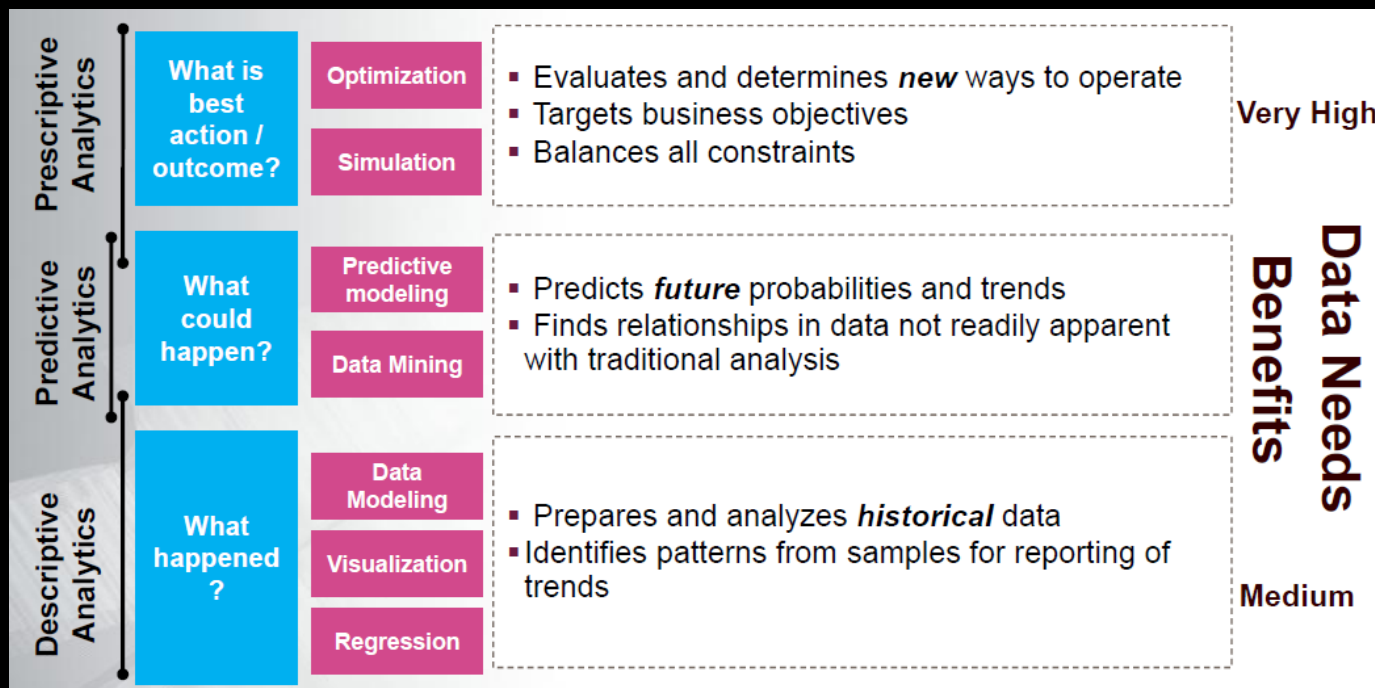
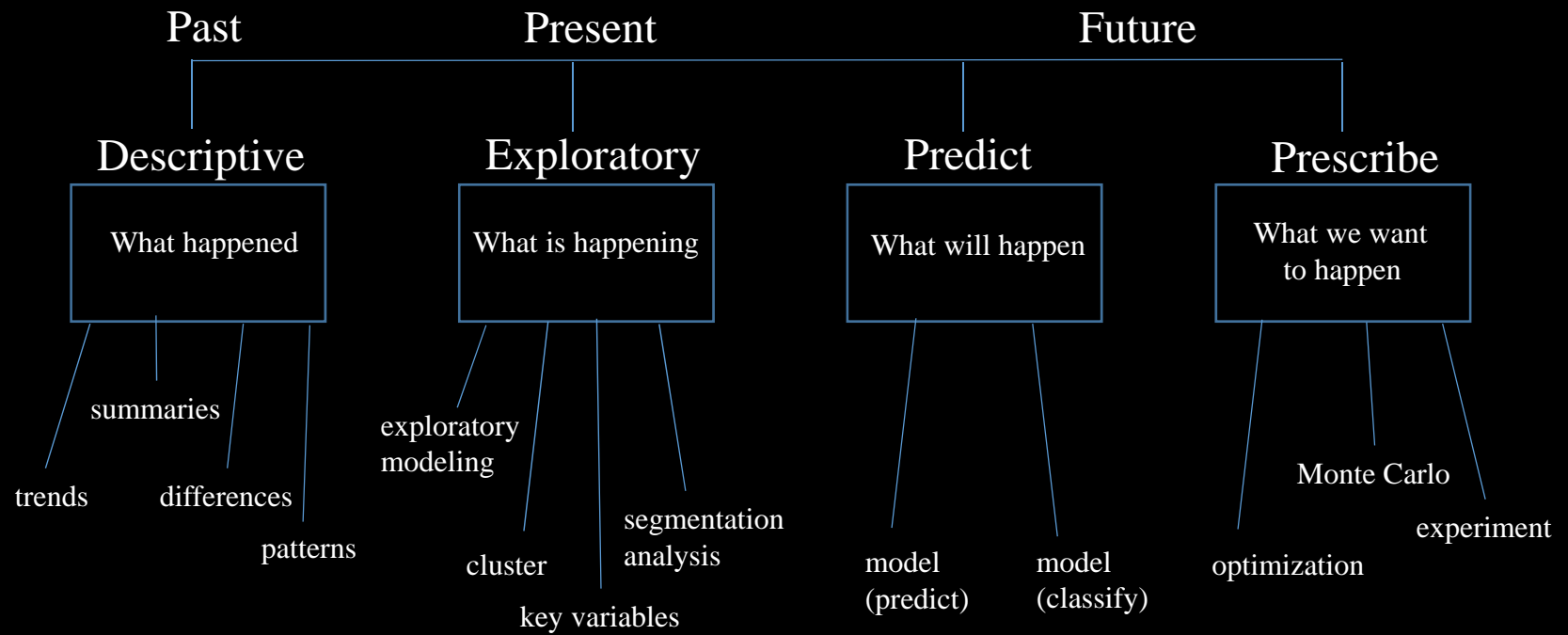
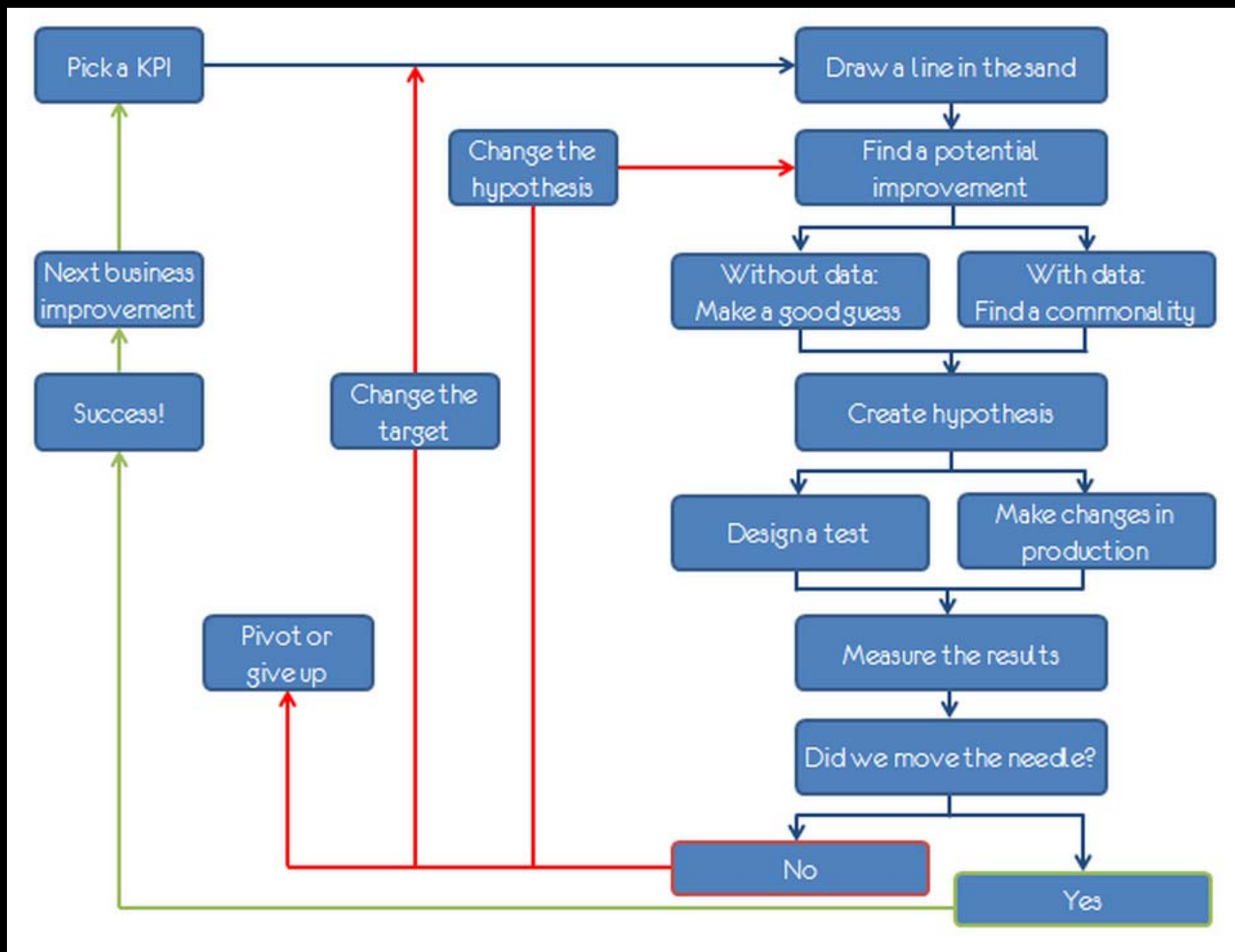


# Differences





Source: Jim Grayson, Ph.D. and Mia Stephens



Avinash Kaushik on Lean Analytics | <http://www.kaushik.net/avinash/lean-analytics-cycle-metrics-hypothesis-experiment-act/>

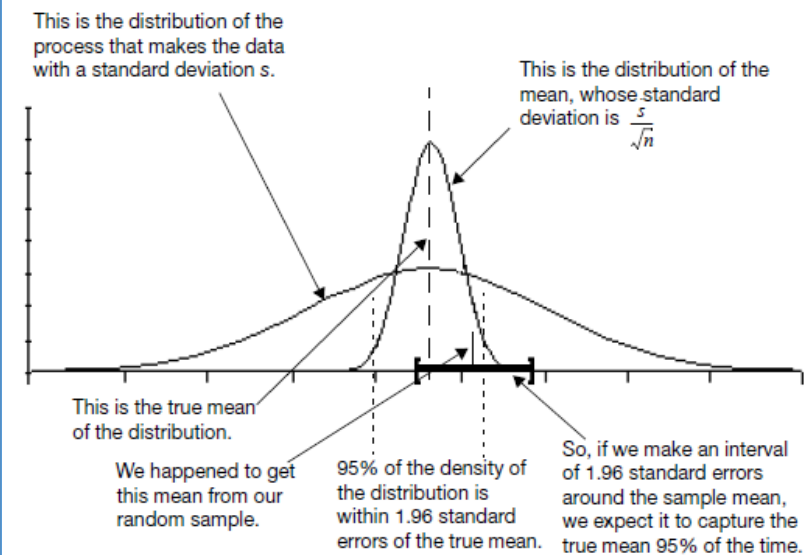
# One Variable

Mean: Hypothesis Testing

Mean: Confidence Interval

# Understanding CIs

Figure 7.10 Illustration of Confidence Interval



JMP Start Statistics, 4e (SAS Press)

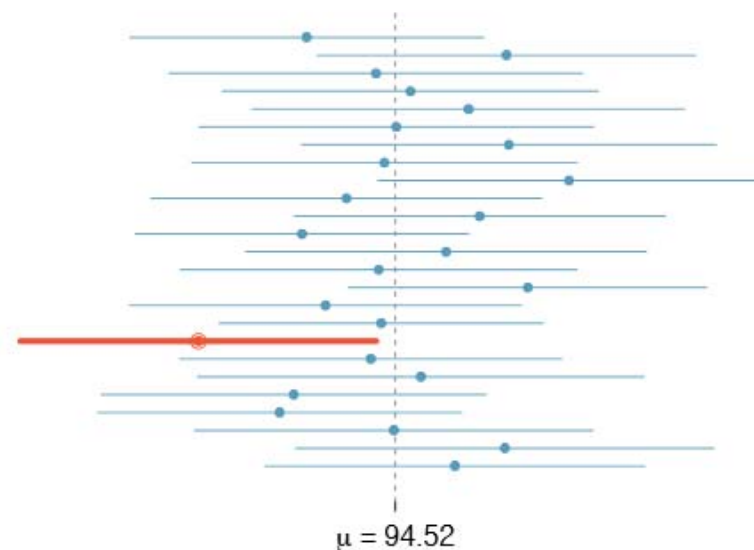


Figure 4.8: Twenty-five samples of size  $n = 100$  were taken from the `run10` data set. For each sample, a confidence interval was created to try to capture the average 10 mile time for the population. Only 1 of these 25 intervals did not capture the true mean,  $\mu = 94.52$  minutes.

OpenIntro Statistics, 2e

Developed by Jim Grayson, PhD

# Confidence Interval Principle

Every confidence interval is constructed as:

Point Estimate  $\pm$  Confidence Level Multiplier \* Standard Error

Every confidence interval is attempting to “capture” the population parameter and is interpreted as:

*We are X% confident [insert confidence level] the true (but unknown) population parameter is between [lower CI limit] and [upper CI limit]*

## One Sample t-Test and CI

### Confidence Interval for the Mean

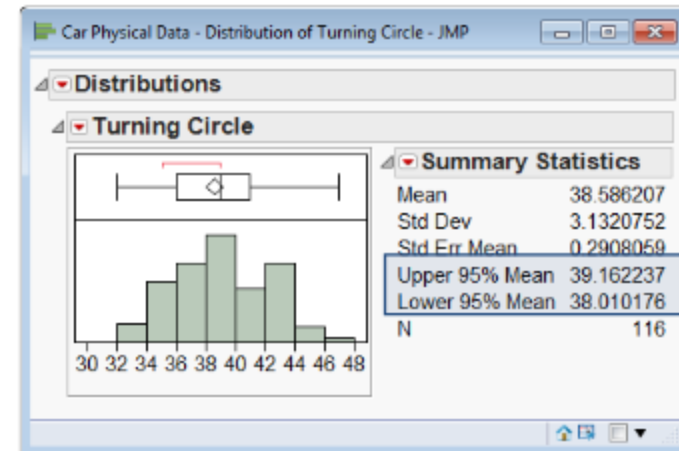
1. From an open JMP® data table, select **Analyze > Distribution**.
2. Select one or more continuous variables from **Select Columns**, click **Y, Columns** (continuous variables have blue triangles), and click **OK**.

The **Upper 95% Mean** and **Lower 95% Mean** give the 95% confidence interval for the true mean (39.163 and 38.01).

Tips:

- To change the display from vertical to horizontal (as shown), click on the **top red triangle** and select **Stack**.
- To change the confidence level, request a one-sided confidence limit or specify sigma, click on the **red triangle** for the variable, select **Confidence Interval**, and select the confidence level or click **Other**.

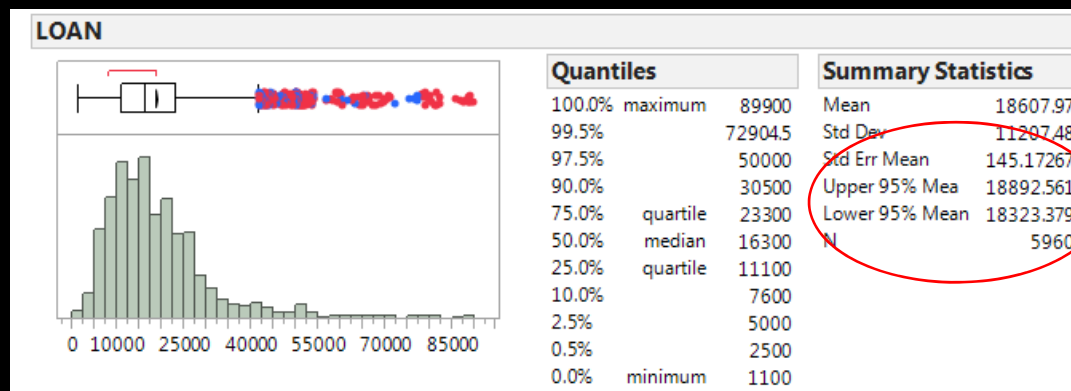
Example: Car Physical Data.jmp (Help > Sample Data)





# Confidence Interval

Source: Equity Data | Distribution Platform



**Red Triangle: Confidence Interval | 0.95**

$$(1 - \alpha) \text{ C.I. for the mean} = \bar{x} \pm \left( t_{1 - \frac{\alpha}{2}} \cdot s_y \right)$$

JMP Start Statistics, 4e (SAS Press)

# One Variable

Mean: Hypothesis Testing

Mean: Confidence Interval

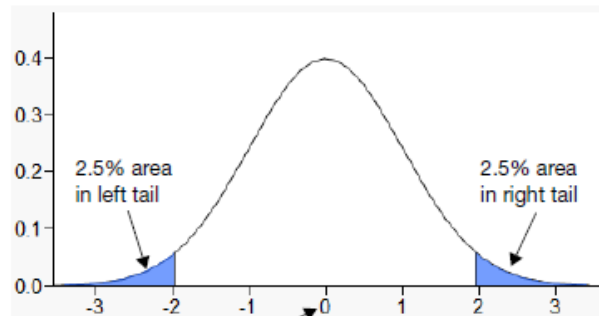
# Hypothesis Test Principles

Every hypothesis test has these elements:

1. There is a null ( $H_0$ ) and alternate ( $H_a$ ) hypothesis – since the null cannot be proved we put what we hope to find in the alternate hypothesis. The null hypothesis is always a hypothesis of “no difference”
2. There is a test statistic (the particular form depends on our question and data) – this is needed because we need a way of deciding between our hypotheses
3. We construct (compute) the test statistic
4. We compare the value of the test statistic against a reference distribution (this reference distribution depends on our data type and our test statistic)
5. This comparison of the test statistic to the reference distribution gives us a probability (called a p-value) of seeing this value for the test statistic when the null hypothesis is indeed true
6. We compare this p-value against some predetermined threshold called alpha (or cut off) and if the p-value is less than alpha we state our conclusion of rejecting the null and accepting the alternate
7. Stated conclusions will be one of two statements:
  - a. We reject the null and accept the alternate recognizing we have a “p-value” chance of being wrong in our conclusion – this is called a Type 1 error
  - b. We fail to reject the null

# Test Distribution

Figure 7.12 Illustration of the Two-Tailed z-test



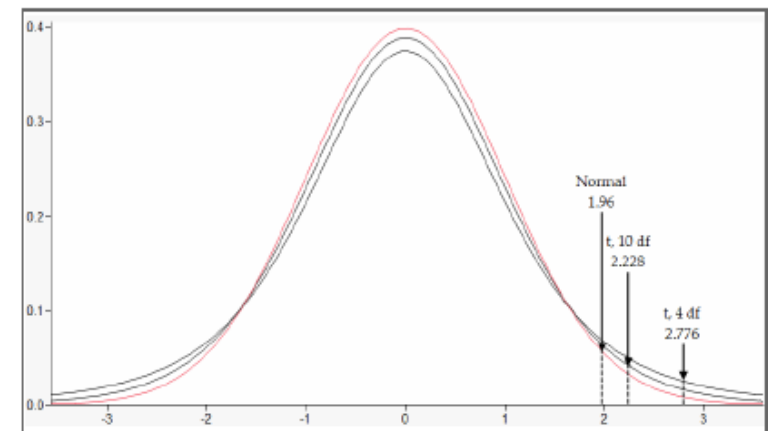
The null hypothesis is that the test statistic has mean 0 and standard deviation 1.

So, if the test statistic falls in one of the extreme regions, and the null hypothesis is true, you have a rare event that happens only 5% of the time. We therefore tend to doubt the null hypothesis.

$$z\text{-statistic} = \frac{\text{estimate} - \text{hypothesized estimate}}{\text{standard deviation}}$$

JMP Start Statistics, 4e (SAS Press)

Figure 7.14 Comparison of Normal and Student's *t* Distributions



$$t\text{-statistic} = \frac{\text{sample mean} - \text{hypothesized value}}{\text{standard error of the mean}}$$

JMP Start Statistics, 4e (SAS Press)

Developed by Jim Grayson, PhD

## One Sample t-Test and CI

### One Sample t-Test for the Mean

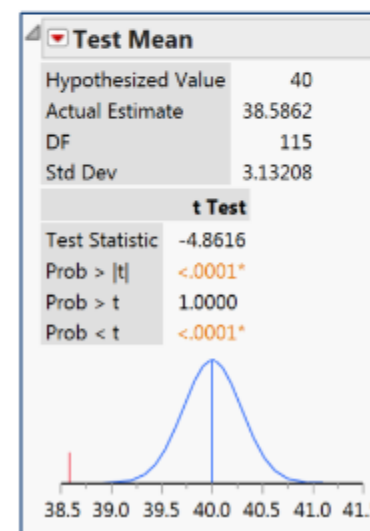
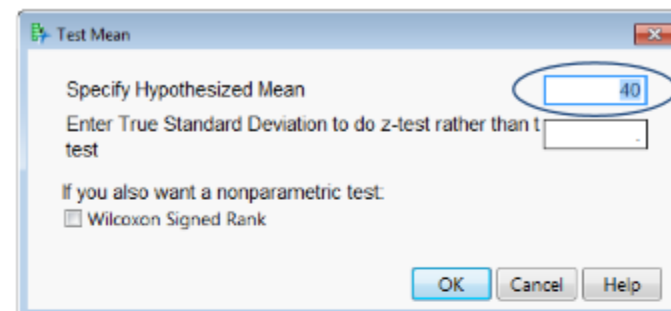
1. From the Distributions report window (shown above), click on the **red triangle** for the variable and select **Test Mean**.
2. Enter the hypothesized value under **Specify Hypothesized Mean**, and click **OK**.

JMP will generate:

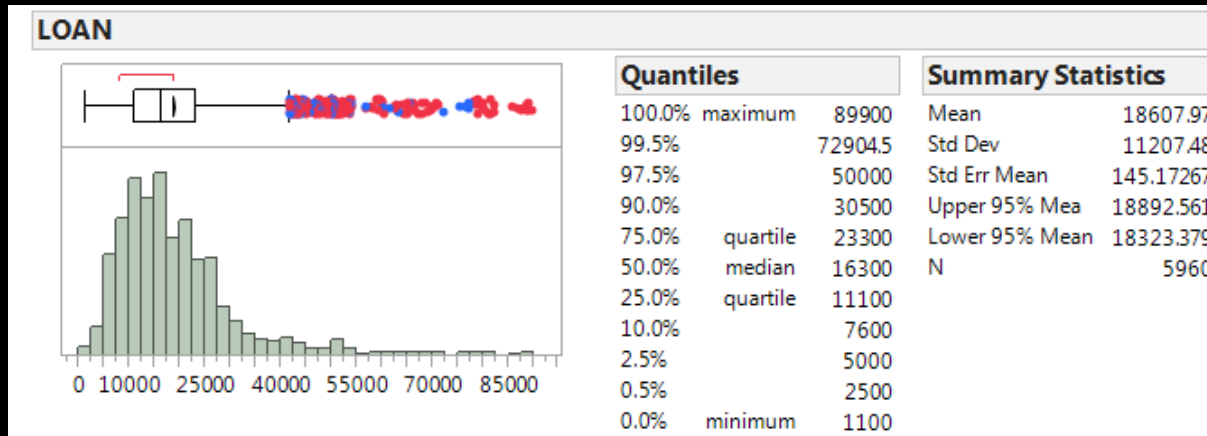
- The **t-Ratio** (next to **Test Statistic**).
- **P-values** for the two-tailed and one-tailed tests.
- A graph to aid in interpreting the p-values, showing the hypothesized mean (center of the curve) and the sample mean (red line).

Interpretation of p-values for this example (using a significance level of 0.05):

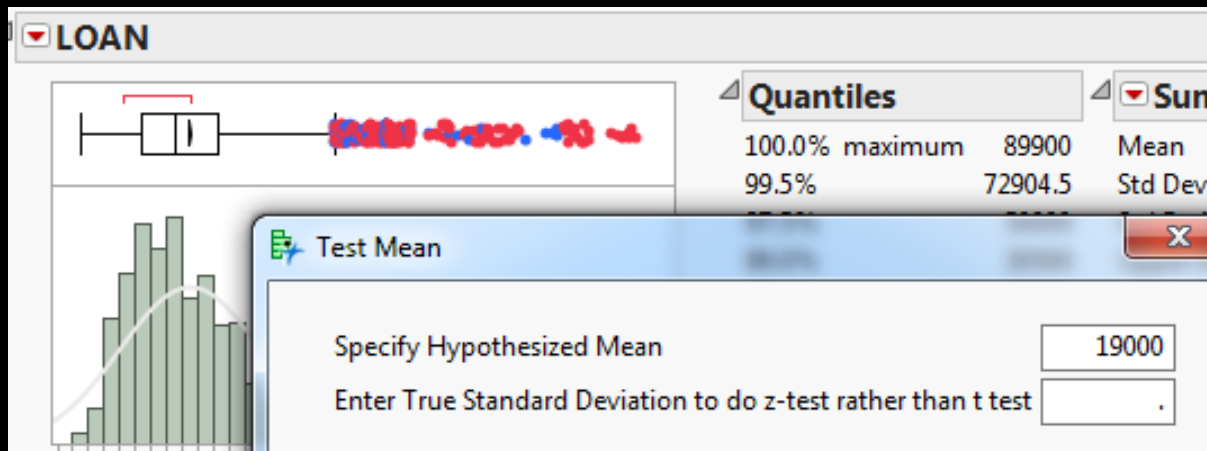
1. **Prob > |t| is less than 0.05 - reject the null hypothesis** that the true mean is 40. This is the two-tailed test. Conclude that the true mean is not 40.
2. **Prob > t is greater than 0.05 - fail to reject the null hypothesis** that the true mean is  $\leq 40$ . This is a one-tailed test. There is insufficient evidence to reject the null hypothesis.
3. **Prob < t is less than 0.05 - reject the null hypothesis** that the true mean is  $\geq 40$ . Conclude that the true mean is less than 40.



# Testing Mean (HT)



Red Triangle: Test Mean | supply value

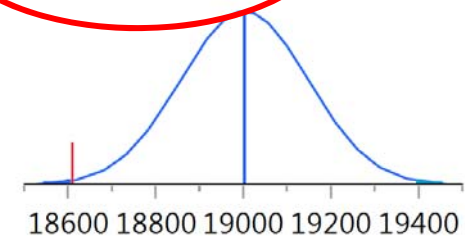


## Test Mean

Hypothesized Value 19000  
 Actual Estimate 18608  
 DF 5959  
 Std Dev 11207.5

## t Test

Test Statistic -2.7004  
 Prob > |t| 0.0069\*  
 Prob > t 0.9965  
 Prob < t 0.0035\*



# Insurance Profits

Data set: Insurance Profits\_GE11 | Source: Business Statistics 3e

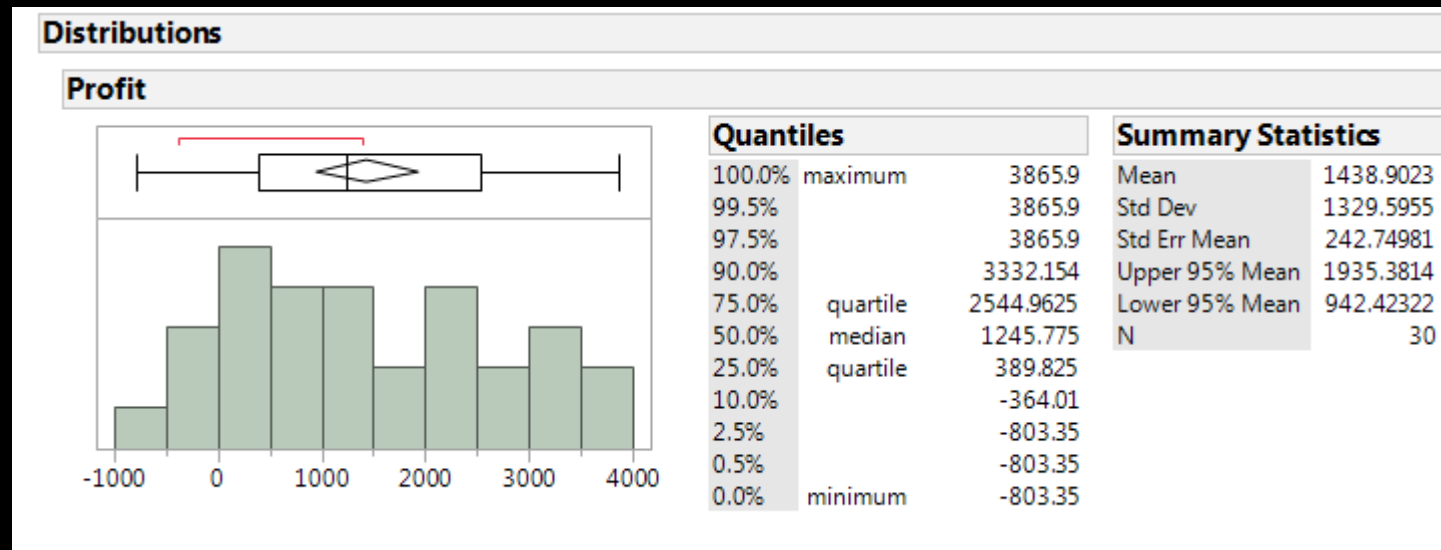
Profit	
222.800	
1756.23	2447.50
1100.85	1847.50
3340.66	865.400
1006.50	1415.65
445.500	2756.94
3255.60	2089.40
3701.85	2692.75
-803.35	2495.70
3865.90	2172.70
463.350	3249.65
-66.200	-397.10
57.9000	-397.31
833.950	186.250
1390.70	590.850
	578.950

A manager wanted to see how well one of her sales representatives was doing, selected a sample of 30 policies and wants you to construct a 95% CI for the mean profit of the policies sold by this sales rep

# Insurance Profits

Data set: Insurance Profits\_GE11 | Source: Business Statistics 3e

Profit	
222.800	2447.50
1756.23	1847.50
1100.85	1847.50
3340.66	865.400
1006.50	1415.65
445.500	2756.94
3255.60	2089.40
3701.85	2692.75
-803.35	2495.70
3865.90	2172.70
463.350	3249.65
-66.200	-397.10
57.9000	-397.31
833.950	186.250
1390.70	590.850
	578.950





# Insurance Profits

Data set: Insurance Profits\_GE11 | Source: Business Statistics 3e

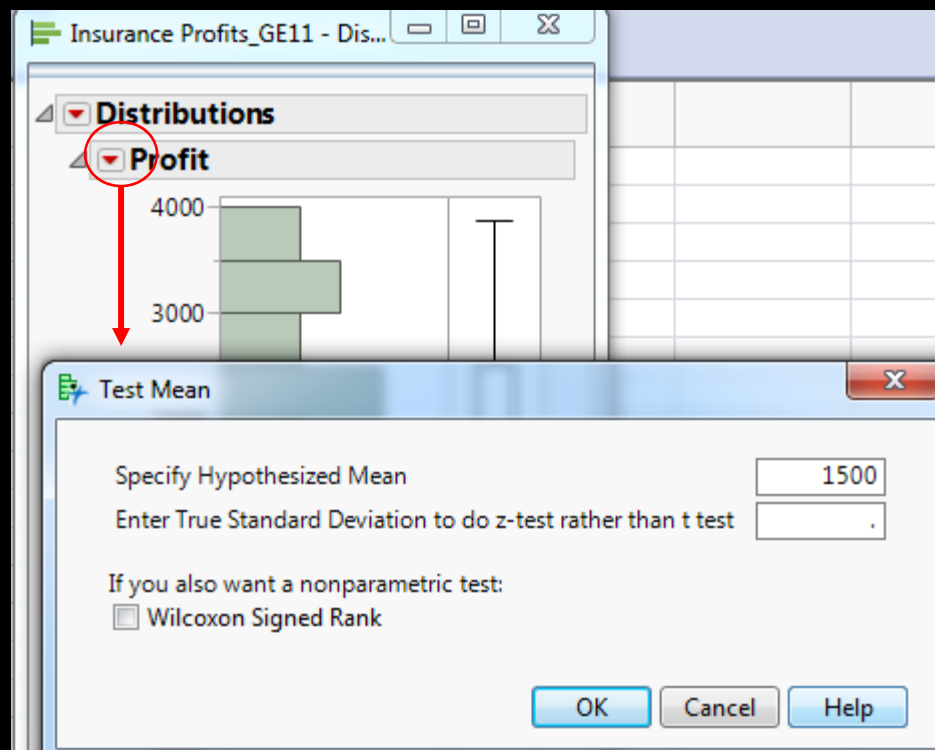
Profit	
222.800	
1756.23	2447.50
1100.85	1847.50
3340.66	865.400
1006.50	1415.65
445.500	2756.94
3255.60	2089.40
3701.85	2692.75
-803.35	2495.70
3865.90	2172.70
463.350	3249.65
-66.200	-397.10
57.9000	-397.31
833.950	186.250
1390.70	590.850
	578.950

A manager wanted to know if there's evidence that the mean profit of policies sold by this sales rep is less than \$1500

# Insurance Profits

Data set: Insurance Profits\_GE11 | Source: Business Statistics 3e

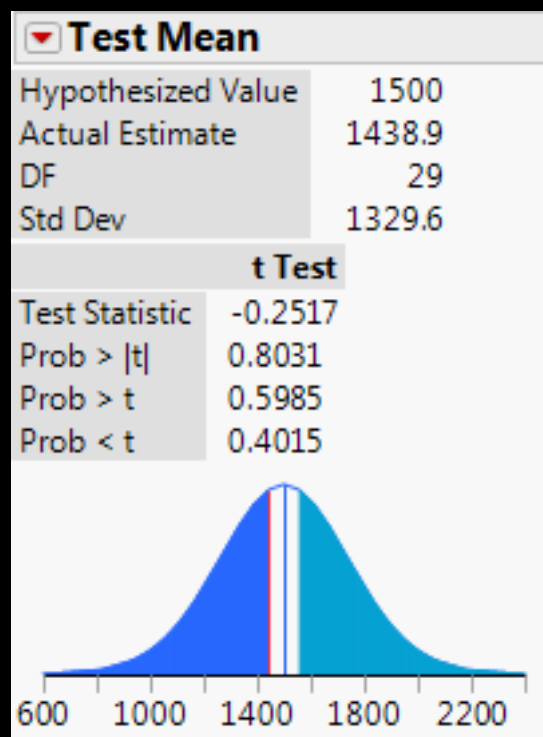
Profit	
222.800	2447.50
1756.23	1847.50
1100.85	865.400
3340.66	1415.65
1006.50	2756.94
445.500	2089.40
3255.60	2692.75
3701.85	2495.70
-803.35	2172.70
3865.90	3249.65
463.350	-397.10
-66.200	-397.31
57.9000	186.250
833.950	590.850
1390.70	578.950



# Insurance Profits

Data set: Insurance Profits\_GE11 | Source: Business Statistics 3e

Profit	
222.800	
1756.23	2447.50
1100.85	1847.50
3340.66	865.400
1006.50	1415.65
445.500	2756.94
3255.60	2089.40
3701.85	2692.75
-803.35	2495.70
3865.90	2172.70
463.350	3249.65
-66.200	-397.10
57.9000	-397.31
833.950	186.250
1390.70	590.850
	578.950



There is not enough evidence in this sample of policies to indicate that the true mean is below \$1500



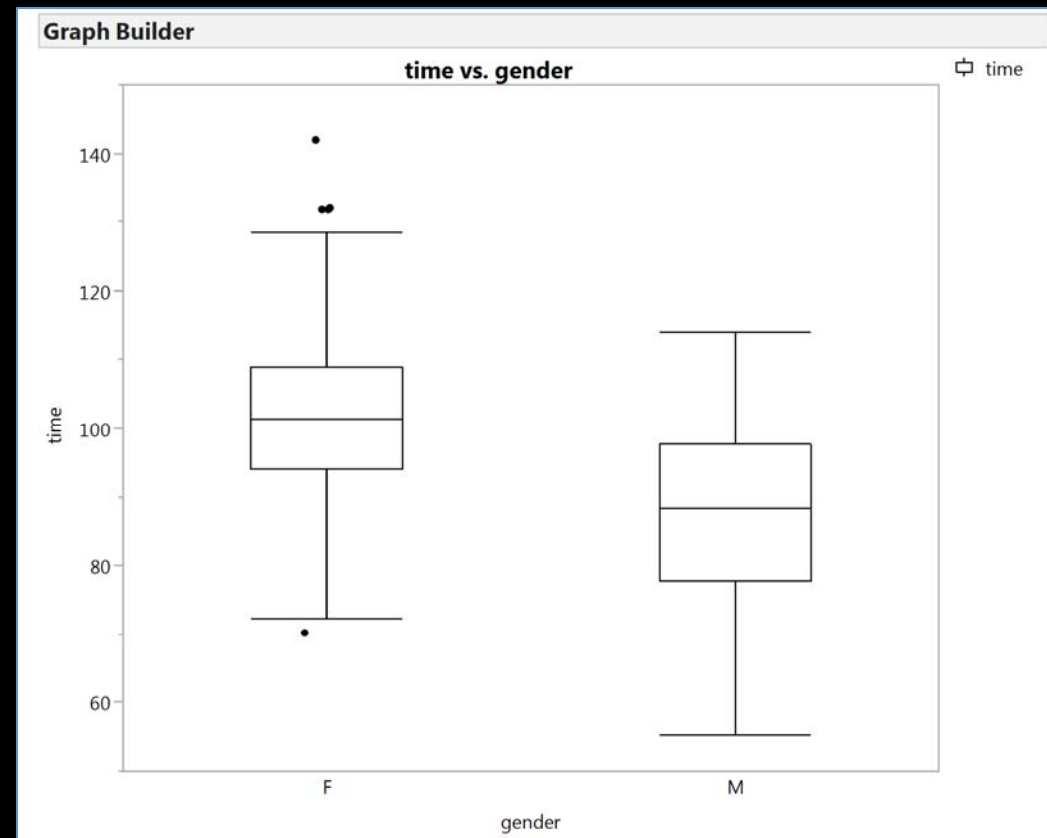
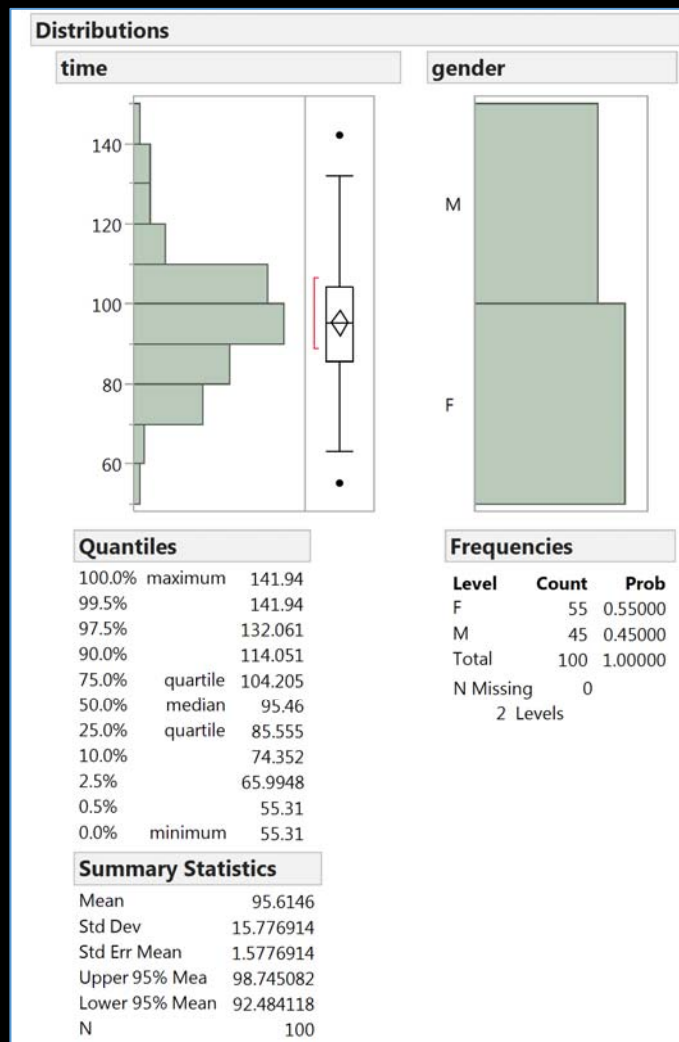
# Two Variable: Continuous Response and Categorical Factor

Difference Between Two Means

- Hypothesis Testing
- Confidence Interval

We would like to estimate the average difference in run times for men and women using the run10Samp data set, which was a simple random sample of 45 men and 55 women from all runners in the 2012 Cherry Blossom Run.

Data source: run10Samp (openintro stats, 2e)

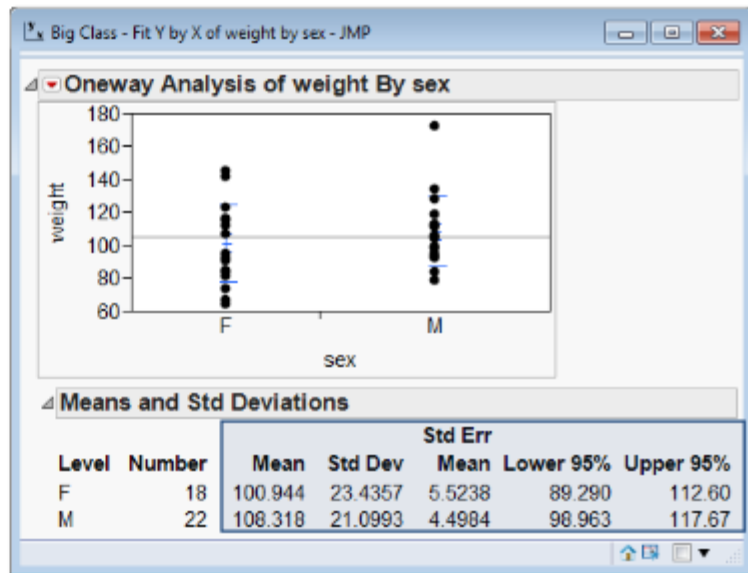


## Two Sample t-Test and CIs

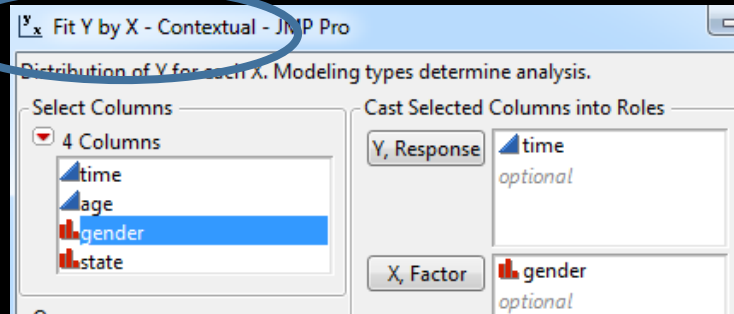
### Confidence Intervals

1. Select **Analyze > Fit Y by X**.
2. Click on a continuous variable from **Select Columns**, and click **Y, Response** (continuous variables have blue triangles).
3. Click on a two-level categorical variable and click **X, Factor** (categorical variables have red or green bars).
4. Click **OK**. The Oneway Analysis output window will display.
5. Click on the **red triangle**, and select **Means and Std Dev** to produce summary statistics and individual confidence intervals for each mean (Lower 95% and Upper 95%).

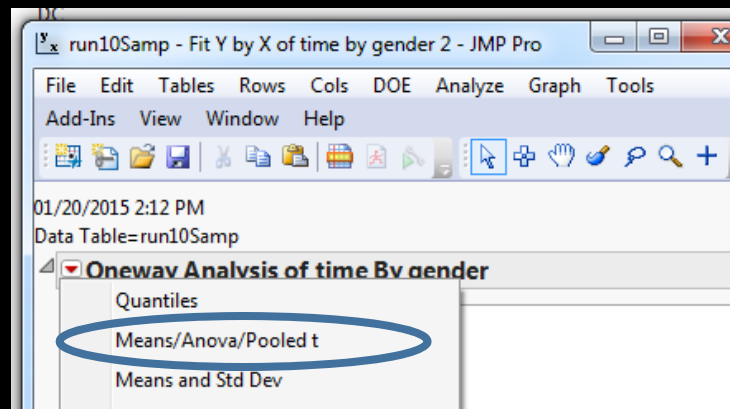
Example: Big Class.jmp (Help > Sample Data)



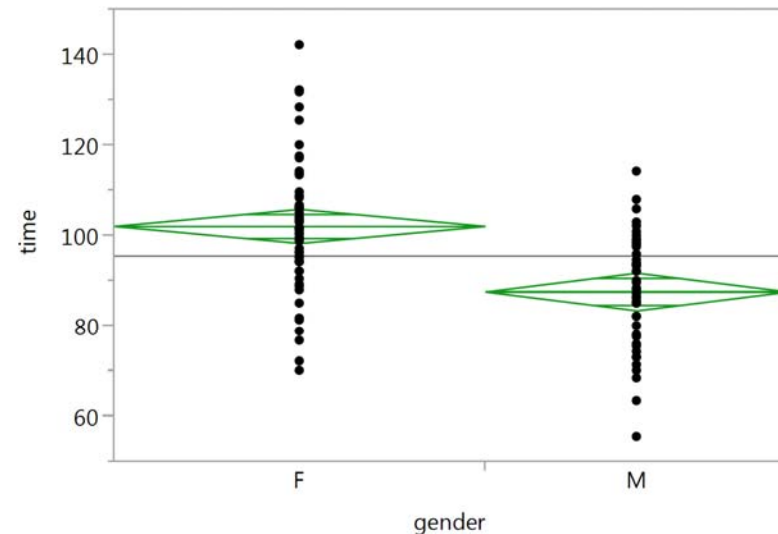
## Analyze | Fit Y by X



## Means/Anova/Pooled t



### Oneway Analysis of time By gender



### Oneway Anova

#### Summary of Fit

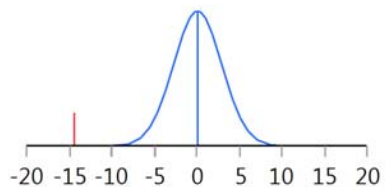
Rsquare	0.210866
Adj Rsquare	0.202814
Root Mean Square Error	14.08646
Mean of Response	95.6146
Observations (or Sum Wgts)	100

#### t Test

M-F

Assuming equal variances

Difference	-14.490	t Ratio	-5.11731
Std Err Dif	2.831	DF	98
Upper CL Dif	-8.871	Prob >  t	<.0001*
Lower CL Dif	-20.109	Prob > t	1.0000
Confidence	0.95	Prob < t	<.0001*





# Two Variable: Continuous Response and Categorical Factor

Difference Between Two Means

- Hypothesis Testing
- Confidence Interval

## Two Sample t-Test and CIs

### Two Sample t-Test

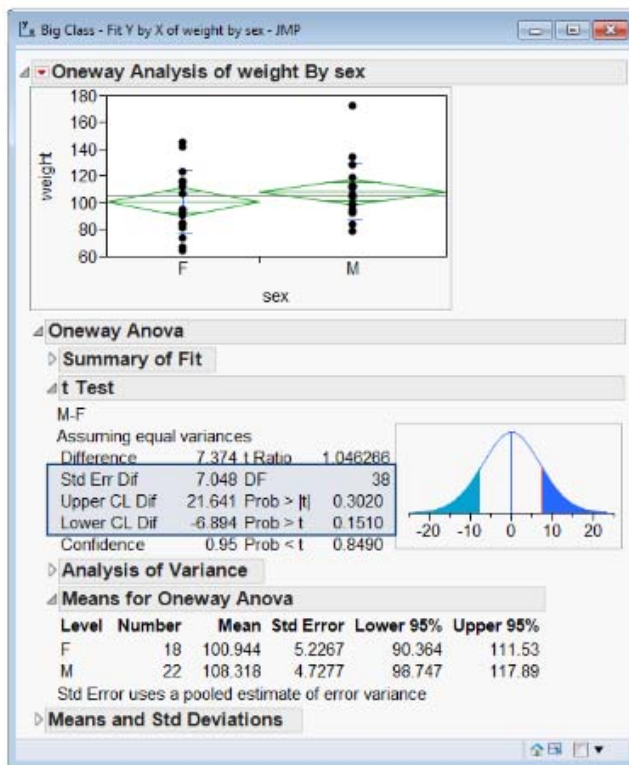
From the Oneway Analysis output window (shown above), click on the **red triangle** and select **Means/Anova/Pooled t**.

JMP® will plot **means diamonds (95% confidence intervals for each mean)**, and will generate:

- The Summary of Fit (not shown).
- The t-test report, with a graph to aid in interpreting the results.
- The Analysis of Variance table.
- Means for Oneway Anova (not shown), which includes confidence intervals based on the pooled estimate of the standard error.

Interpretation of the results (using a significance level of 0.05 - click the **red triangle**, Set  $\alpha$  Level to change significance level):

1. **Upper CL Dif** and **Lower CL Dif** give the 95% CI for the true difference. Since the 95% CI contains zero, **conclude that there is not a significant difference** between the means.
2. **Prob > |t|** is the p-value for the two-tailed test. The null hypothesis is that means are equal (the mean difference is zero). Since the **Prob > |t|** is **greater than 0.05**, **cannot reject the null hypothesis** (i.e., we cannot conclude that there is a significant difference).



Notes: **Means/Anova/Pooled t** is the test under the assumption of equal variances. For a test without the assumption of equal variances, select **t Test** instead. See the **Basic Analysis** book (under **Help > Books**) for more details.

## t Test

M-F

Assuming equal variances

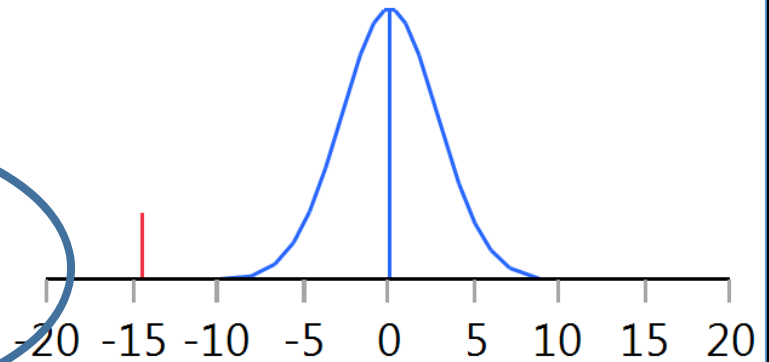
Difference -14.490 t Ratio -5.11731

Std Err Dif 2.831 DF 98

Upper CL Dif -8.871 Prob > |t| <.0001\*

Lower CL Dif -20.109 Prob > t 1.0000

Confidence 0.95 Prob < t <.0001\*

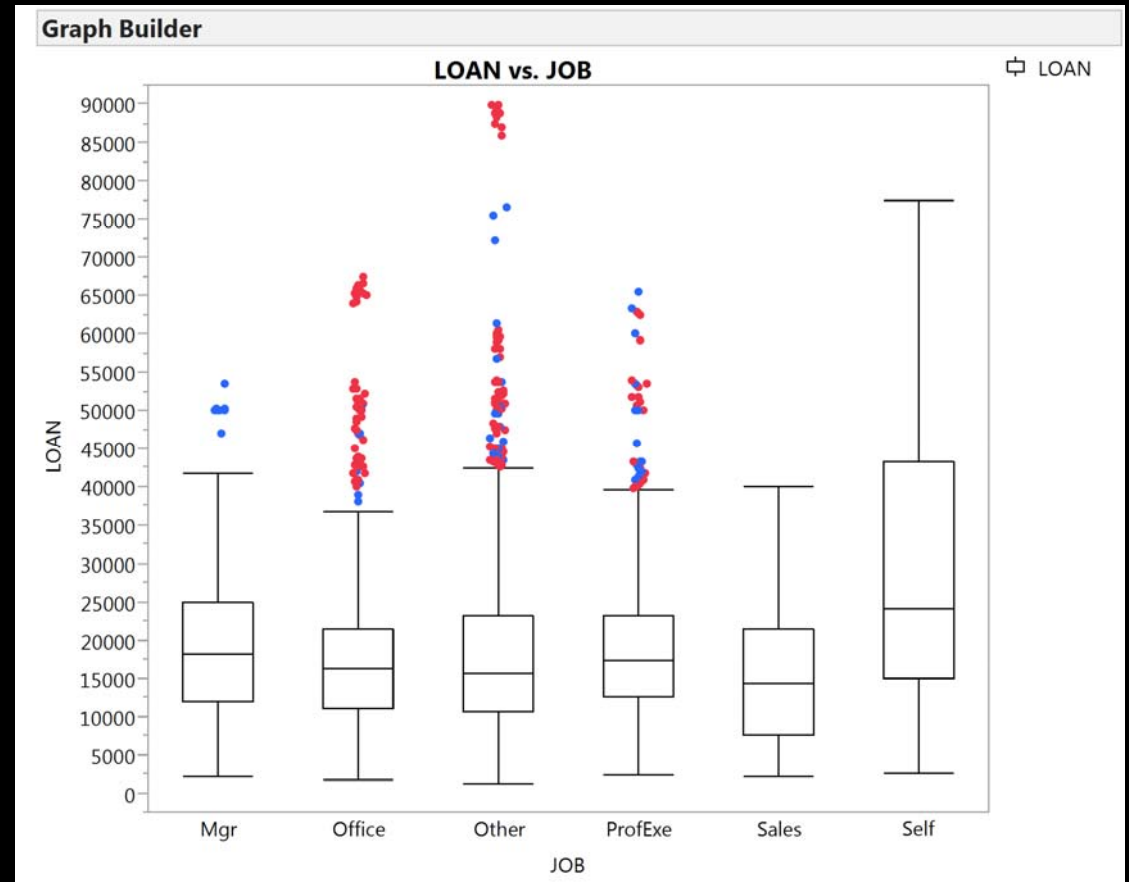
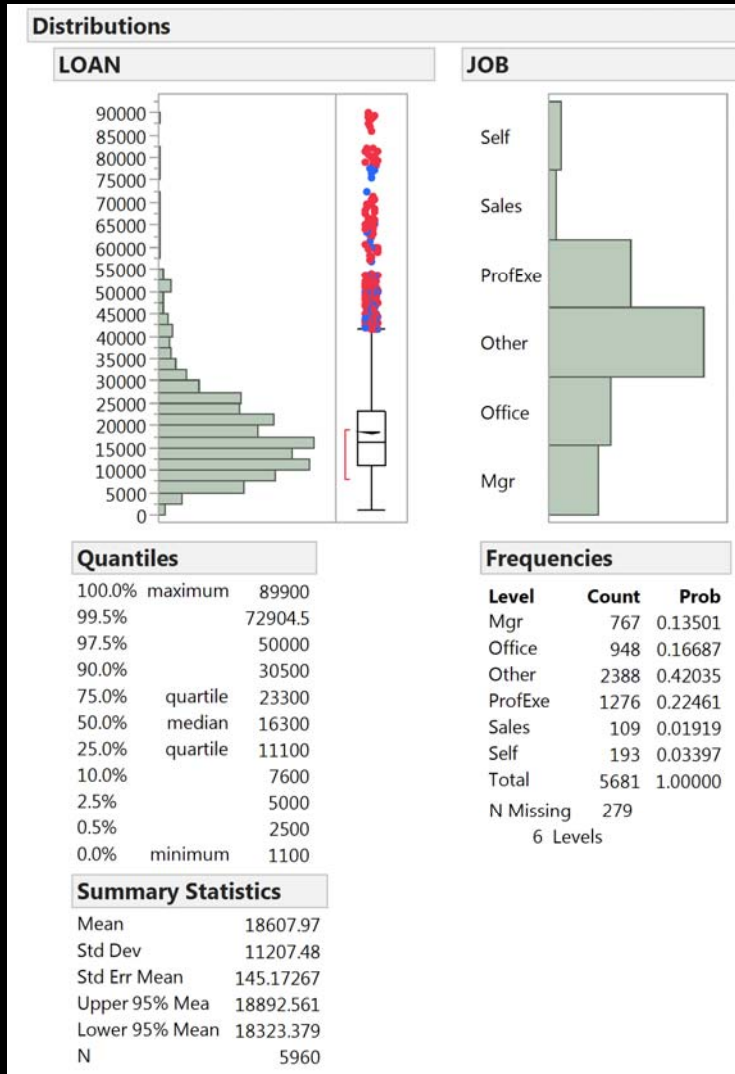


# Multiple Variable: Continuous Response with Categorical Variables

ANOVA

F Test

Source: Equity Data



# One-Way ANOVA

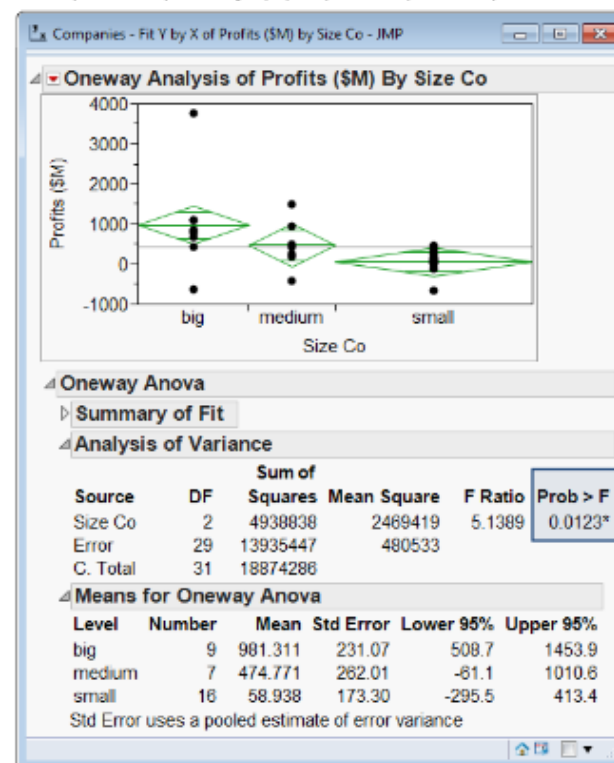
## One-Way Analysis of Variance

1. Select **Analyze > Fit Y by X**.
2. Click on a continuous variable from **Select Columns**, and Click **Y, Response** (continuous variables have blue triangles).
3. Click on a categorical variable and click **X, Factor** (categorical variables have red or green bars).
4. Click **OK**. The Oneway Analysis output window will display.
5. Click on the **red triangle**, and select **Means/Anova**.

JMP® will plot means diamonds (95% confidence intervals for each mean), and will generate:

- The Summary of Fit.
- The Analysis of Variance (Anova) table.
- Means for Oneway Anova, containing summary statistics and confidence intervals for each mean (based on the pooled estimate of the standard error).

Example: Companies.jmp (Help > Sample Data)



Interpretation of the results in the Anova table (using a significance level of 0.05 – click the red triangle, Set  $\alpha$  Level to change significance level):

- The null hypothesis is that there are no differences between the population means (i.e., all means are equal).
- **Prob > F** is the p-value for the whole model test. Since the **Prob > F** is less than 0.05, reject the null hypothesis. Conclude that there are differences between at least two of the means.
- To determine which means are different, a post hoc multiple comparison technique can be used.

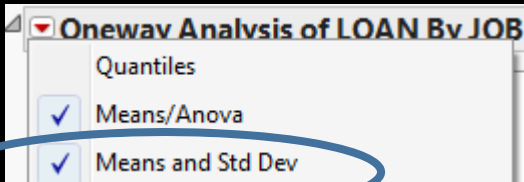
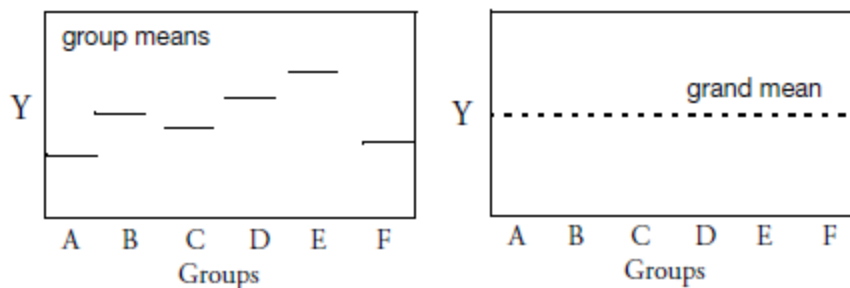


Figure 9.1 Different Mean for Each Group Versus a Single Overall Mean

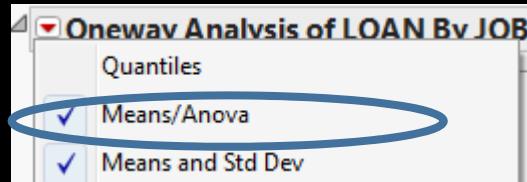


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### Means for Oneway Anova

Level	Number	Mean	Std Error	Lower 95%	Upper 95%
Mgr	767	19155.3	389.2	18392	19918
Office	948	18142.6	350.1	17456	18829
Other	2388	18061.7	220.6	17629	18494
ProfExe	1276	18983.5	301.7	18392	19575
Sales	109	14913.8	1032.4	12890	16938
Self	193	28314.5	775.9	26793	29836

Std Error uses a pooled estimate of error variance



## Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
JOB	5	2.093e+10	4.1861e+9	36.0300	<.0001*
Error	5675	6.5934e+11	116183575		
C. Total	5680	6.8027e+11			

$H_0$ : The mean outcome is the same across all groups. In statistical notation,  $\mu_1 = \mu_2 = \dots = \mu_k$  where  $\mu_i$  represents the mean of the outcome for observations in category  $i$ .

$H_A$ : At least one mean is different.

OpenIntro Statistics, 2e

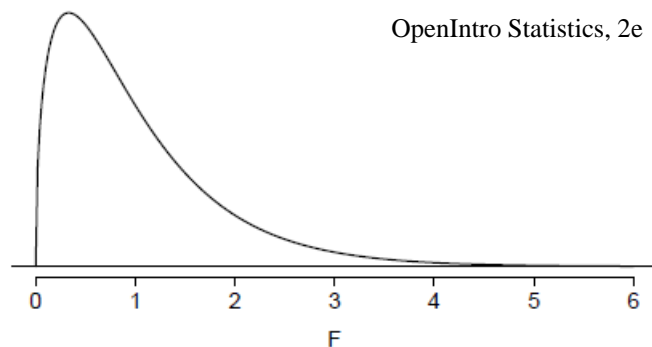


Figure 5.29: An  $F$  distribution with  $df_1 = 3$  and  $df_2 = 323$ .



## One-Way ANOVA

### Multiple Comparison Procedures

From the Oneway Analysis output window (shown above), click on the **red triangle**, select **Compare Means**, and select one of the four methods (described in JMP Help).

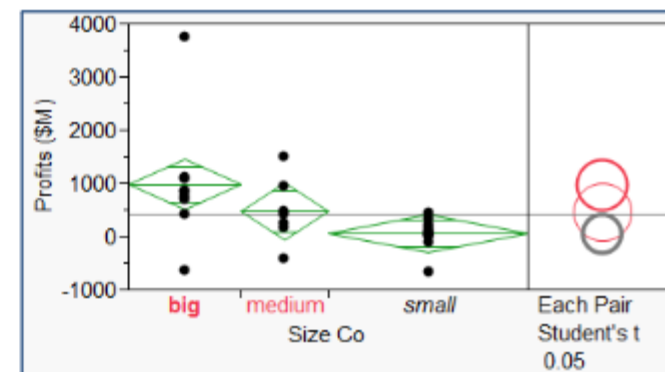
**Each Pair, Student's t** has been selected. This produces comparison circles (shown), along with statistical output (not shown).

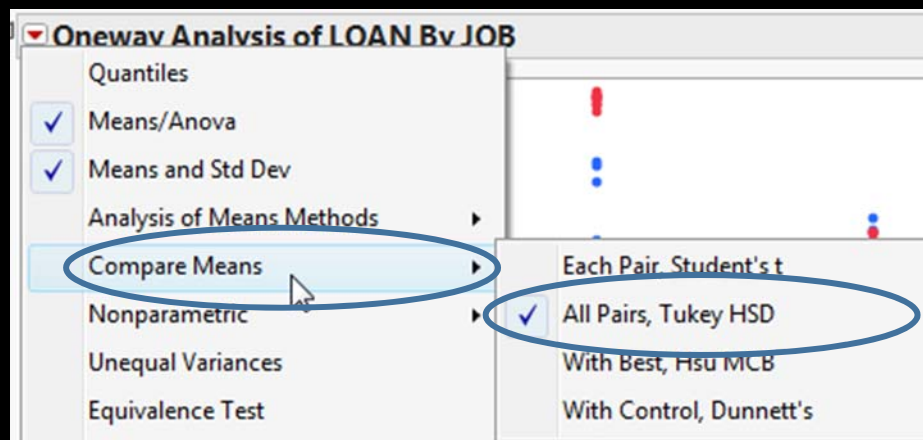
Click on a circle for a mean to test for paired differences.

- The **selected mean** will have a **bold, red circle and variable label**.
- Means that are **not significantly different** from the selected mean will have **unbolded, red circles and variable labels**.
- Means that are **significantly different** from the selected mean will have **gray circles and gray italicized variable labels**.

In this example, the mean for **big** is significantly different from the mean for **small**, but is not significantly different from the mean for **medium**.

Each Pair, Student's t  
All Pairs, Tukey HSD  
With Best, Hsu MCB  
With Control, Dunnett's





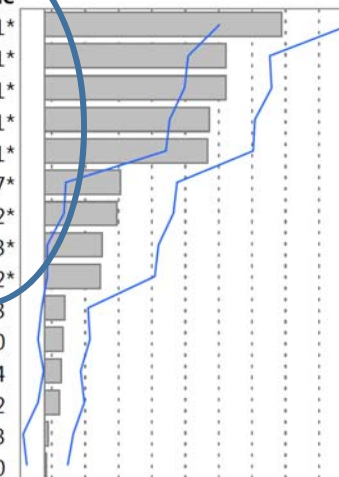
### Connecting Letters Report

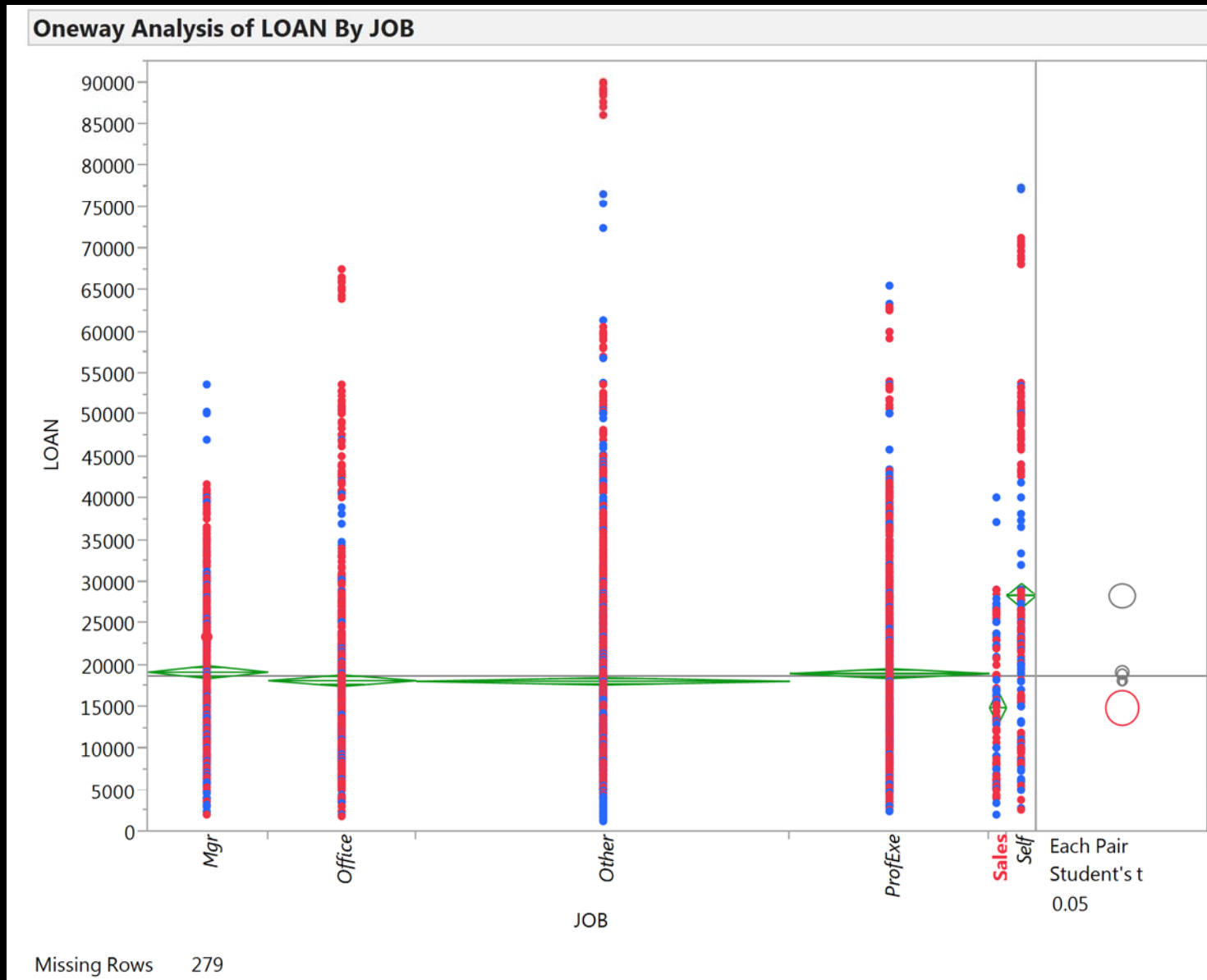
Level	Mean
Self A	28314.508
Mgr B	19155.280
ProfExe B	18983.464
Office B	18142.616
Other B	18061.683
Sales C	14913.761

Levels not connected by same letter are significantly different.

### Ordered Differences Report

Level	- Level	Difference	Std Err Dif	Lower CL	Upper CL	p-Value
Self	Sales	13400.75	1291.469	9719.13	17082.37	<.0001*
Self	Other	10252.82	806.623	7953.37	12557.28	<.0001*
Self	Office	10171.89	851.201	7745.35	12598.43	<.0001*
Self	ProfExe	9331.04	832.490	6957.84	11704.24	<.0001*
Self	Mgr	9159.23	868.024	6684.73	11633.72	<.0001*
Mgr	Sales	4241.52	1103.350	1096.17	7386.86	0.0017*
ProfExe	Sales	4069.70	1075.620	1003.41	7135.99	0.0022*
Office	Sales	3228.85	1090.166	121.10	6336.61	0.0363*
Other	Sales	3147.92	1055.726	138.34	6157.50	0.0342*
Mgr	Other	1093.60	447.360	-181.70	2368.90	0.1413
Mgr	Office	1012.66	523.483	-479.64	2504.97	0.3810
ProfExe	Other	921.78	373.773	-143.74	1987.30	0.1344
ProfExe	Office	840.85	462.179	-476.70	2158.39	0.4532
Mgr	ProfExe	171.82	492.474	-1232.09	1575.72	0.9993
Office	Other	80.93	413.775	-1098.62	1260.49	1.0000





College departments commonly run multiple lectures of the same introductory course each semester because of high demand. Consider a statistics department that runs three lectures of an introductory statistics course. We might like to determine whether there are statistically significant differences in first exam scores in these three classes (A, B, and C). Describe appropriate hypotheses to determine whether there are any differences between the three classes.

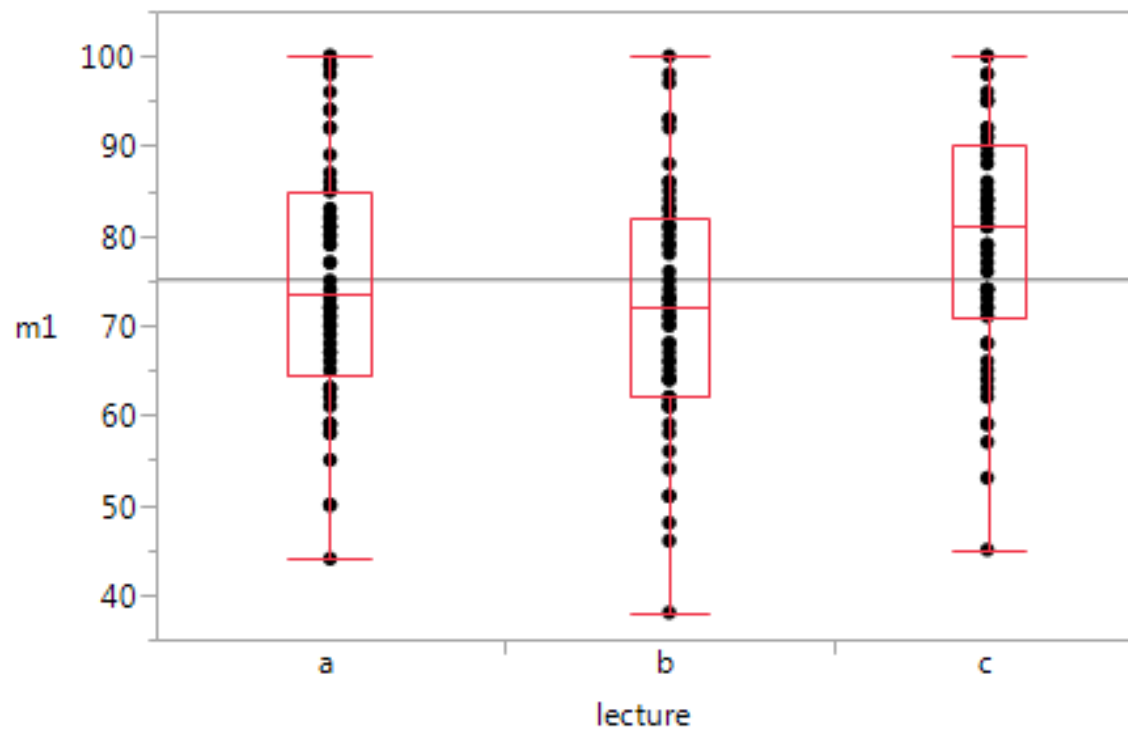
The hypotheses may be written in the following form:

$H_0$ : The average score is identical in all lectures. Any observed difference is due to chance.

$H_A$ : The average score varies by class. We would reject the null hypothesis in favor of the alternative hypothesis if there were larger differences among the class averages than what we might expect from chance alone.

Example 5.34, Open Intro Statistics | Data set: classData

### Oneway Analysis of m1 By lecture



#### Quantiles

Level	Minimum	10%	25%	Median	75%	90%	Maximum
a	44	58	64.5	73.5	85	96.2	100
b	38	52.8	62	72	82	92.4	100
c	45	59.6	71	81	90	95.8	100

## Oneway Anova

### Summary of Fit

Rsquare	0.041482
Adj Rsquare	0.029575
Root Mean Square Error	13.60721
Mean of Response	75.2439
Observations (or Sum Wgts)	164

### Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
lecture	2	1290.114	645.057	3.4839	0.0330*
Error	161	29810.130	185.156		
C. Total	163	31100.244			

### Means for Oneway Anova

Level	Number	Mean	Std Error	Lower 95%	Upper 95%
a	58	75.1034	1.7867	71.575	78.632
b	55	71.9636	1.8348	68.340	75.587
c	51	78.9412	1.9054	75.178	82.704

Std Error uses a pooled estimate of error variance

## Means Comparisons

### Comparisons for all pairs using Tukey-Kramer HSD

#### Confidence Quantile

q*	Alpha
2.36563	0.05

#### HSD Threshold Matrix

Abs(Dif)-HSD

	c	a	b
c	-6.3745	-2.3414	0.7200
a	-2.3414	-5.9775	-2.9186
b	0.7200	-2.9186	-6.1383

Positive values show pairs of means that are significantly different.

#### Connecting Letters Report

Level	Mean
c A	78.941176
a A B	75.103448
b B	71.963636

Levels not connected by same letter are significantly different.

#### Ordered Differences Report

Level	- Level	Difference	Std Err Dif	Lower CL	Upper CL	p-Value
c	b	6.977540	2.645182	0.72001	13.23507	0.0247*
c	a	3.837728	2.612060	-2.34145	10.01690	0.3084
a	b	3.139812	2.561019	-2.91862	9.19824	0.4396

